

PROGRESSION THROUGH CALCULATIONS FOR DIVISION

Overview

This document is set out in stages rather than in year groups to take account for children's different rates of learning. The following extract comes from the Mathematics Programme of Study: Key Stages 1 and 2, September 2013.

The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study. In addition, schools can introduce key stage content during an earlier key stage, if appropriate.

Chunking is an acceptable method for children to be taught by the end of Year 6 both for short and long division but teachers need to follow the guidance specified within the relevant stages of this document.

For an overview of age related expectations for each year group, look at the attached progression map for \times and \div produced by the NCETM.

Other key points

- New learning is likely to be taught to groups rather than the whole class to acknowledge the different learning stages of the children.
- Children need to understand that division can be sharing or grouping (repeated subtraction).
- Children should understand that, unlike multiplication, division is **not** commutative.
- Ensure that children understand the = sign means is the same as, not makes, and that children see calculations where the equals sign is in a different position, e.g. $12 \div 3 = 4$ and $4 = 12 \div 3$.
- Children should be encouraged to approximate before calculating and check whether their answer is reasonable.

Glossary for division

Dividend (56): the first number in the division, that which is to be divided.

Divisor (7): the number by which the dividend is divided.

Quotient (8): the result of dividing one number by another.

$$56 \div 7 = 8$$

STAGE 1

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities involving equal groups and sharing items using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.



STAGE 2

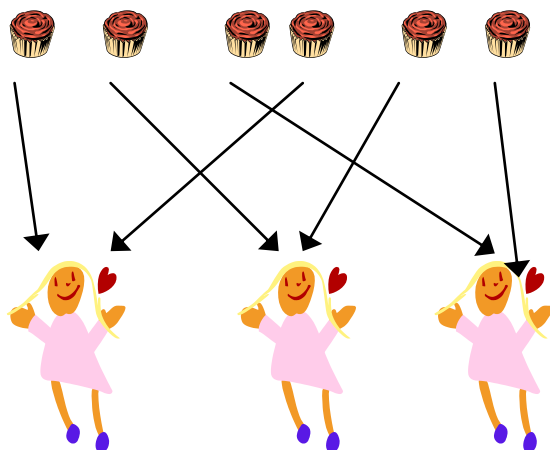
In problem solving contexts, children will use practical equipment to share out objects equally and to group objects to represent division.

Children will develop their understanding of division and use jottings to support calculation

✓ **Sharing equally**

$$6 \div 3 = 2$$

There are 3 children and 6 cakes. How many can they each have, if I share them out equally?

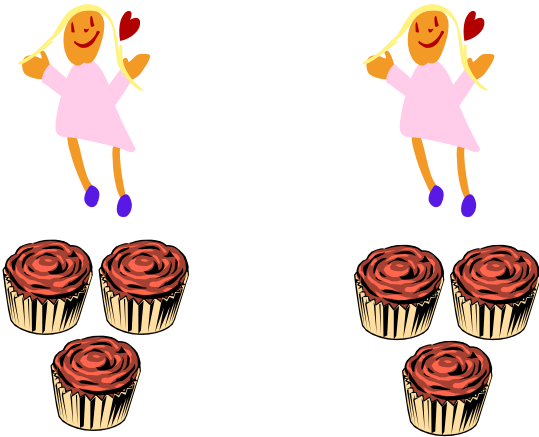


Children should find the answer by counting how many cakes **1 child** has got.

✓ **Grouping**

There are 6 cakes. How many children can have 3 cakes each?

$$6 \div 3 = 2$$



Children should find the answer by counting how many groups of 3 there are.

STAGE 3

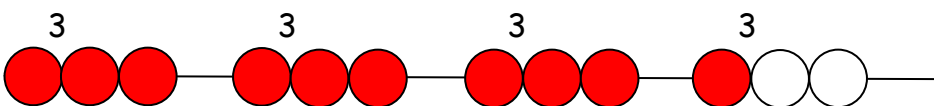
Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

$$12 \div 3 =$$



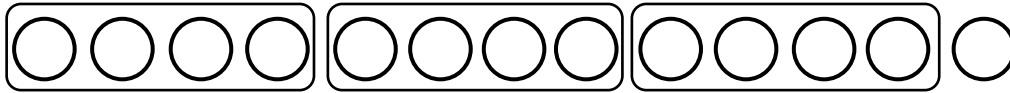
Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

- **Repeated subtraction using bead bar builds on the grouping model**



Children should also move onto calculations involving remainders.

$$13 \div 4 =$$



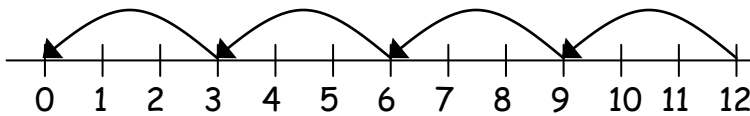
$$13 \div 4 = 3 \text{ remainder } 1$$

STAGE 4

Ensure that the emphasis in Y3 is on grouping rather than sharing.

Repeated subtraction using a number line builds on the grouping model.

$$12 \div 3 = 4$$



Initially start with grouping on a numbered number line.

Numicon is a fantastic resource to use for grouping. The image below shows how the grouping method has been used by a child to answer the calculation $20 \div 4 =$



STAGE 5

Children will continue to use grouping (repeated subtraction) to represent their calculations, answering questions such as:

$$24 \div 2 =$$

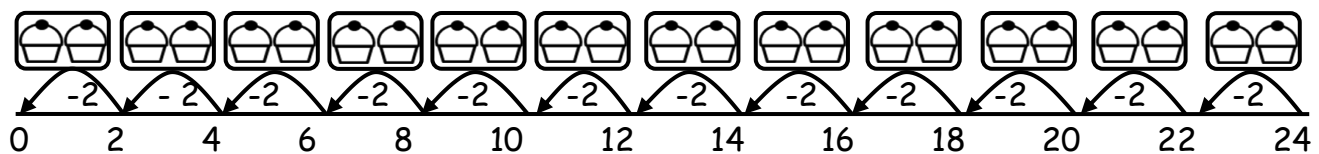
or

There are 24 cupcakes, how many people can have 2 cupcakes each?



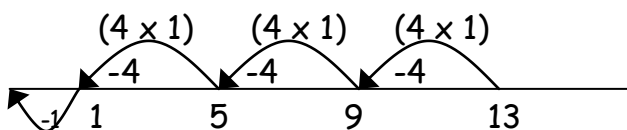
This should also be modelled alongside a number line to emphasise that grouping is repeated subtraction.

$$(2 \times 1) (2 \times 1) (2 \times 1) (2 \times 1) (2 \times 1) (2 \times 1) (2 \times 1) (2 \times 1) (2 \times 1) (2 \times 1) (2 \times 1) (2 \times 1)$$



Children should also move onto calculations involving remainders.

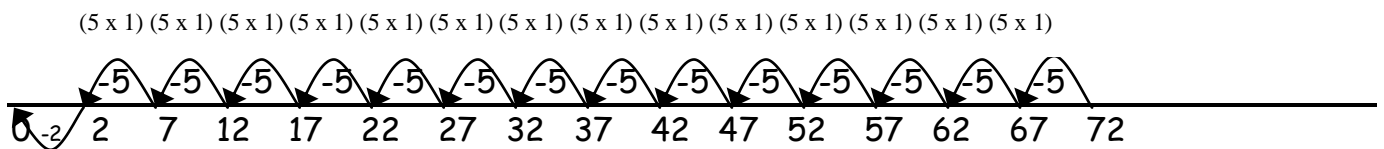
$$13 \div 4 = 3 \text{ r } 1$$



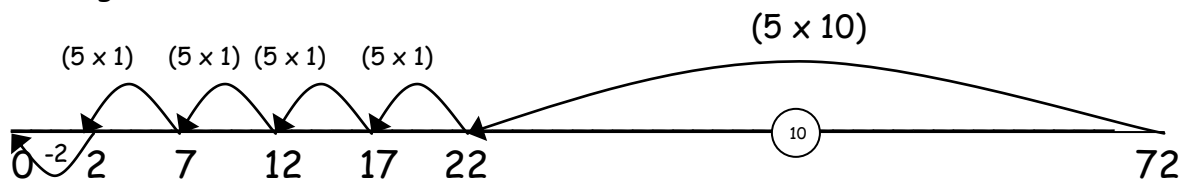
STAGE 6

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s - numbers with which the children are more familiar.

$$72 \div 5 = 14 \text{ r } 2$$



Moving onto:



The 4 groups of 5 could then be subtracted in one chunk to make the above method even more efficient.

Numicon can again be used at this stage for children who require it.

STAGE 7

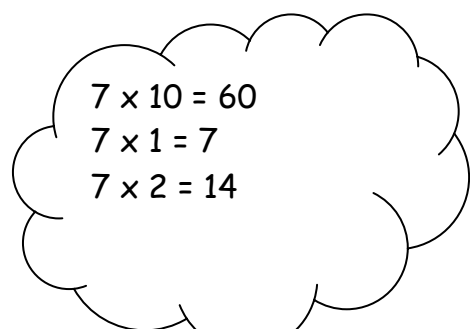
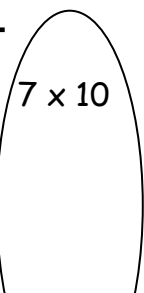
Then onto the vertical 'expanded' method:

Short division $TU \div U$ is the same idea as taking away multiples or "chunks" of the divisor. Pupils, when confident will not need the number line to support these jottings.

Often a thought bubble can help children to prepare the "chunk" they wish to subtract using doubling and halving strategies etc.

$$84 \div 7$$

$$\begin{array}{r} 12 \\ 7 \overline{) 84} \\ \underline{- 70} \end{array}$$



$$\begin{array}{r} 14 \\ - 14 \\ \hline 0 \end{array} \quad 7 \times 2$$

Answer : 12

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

STAGE 8

- 'Short' division of $TU \div U$ can be introduced as a more compact recording of the mental method of partitioning, when pupils are secure at stage 7.

The short division method is recorded like this:

$$\begin{array}{r} 10 + 2 \\ 7 \overline{)70 + 14} \end{array}$$

This is then shortened to:

$$\begin{array}{r} 12 \\ 7 \overline{)8 \text{ } ^1 4} \end{array}$$

The carry digit '1' represents the 1 ten that has been exchanged for 10 ones. In the first recording above it is written in front of the 4 to show that 14 is to be divided by 7. In second it is written as a superscript.

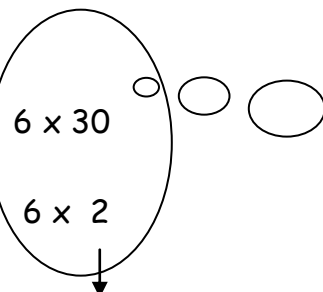
The 12 written above the line represents the answer: 10 + 2, or 1 ten and 2 ones.

STAGE 9

Short division $HTU \div U$

$$196 \div 6$$

$$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{)196} \\ - 180 \\ \hline 16 \\ - 12 \\ \hline 4 \end{array}$$



$$\begin{array}{l} 6 \times 10 = 60 \\ 6 \times 20 = 120 \\ 6 \times 30 = 180 \end{array}$$

Answer : 32 remainder 4 or 32 r 4

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Any remainders should be shown as fractions, i.e. if the children were dividing 352 by 6, the answer should be shown as $58 \frac{4}{6}$ which could then be written as $58 \frac{2}{3}$ in its lowest terms.

STAGE 10

'Short' division of $HTU \div U$ can be introduced as an alternative, more compact recording, when children are secure at stage 9. No chunking is involved since the links are to partitioning, not repeated subtraction.

The short division method is recorded like this:

$$3 \overline{)290 + 1} = 3 \overline{)270 + 21} \begin{matrix} 90 + 7 \\ \end{matrix}$$

This is then shortened to:

$$3 \overline{)2 \overset{9}{7} 1}$$

The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that a total of 21 ones are to be divided by 3.

The 97 written above the line represents the answer: $90 + 7$, or 9 tens and 7 ones.

STAGE 11

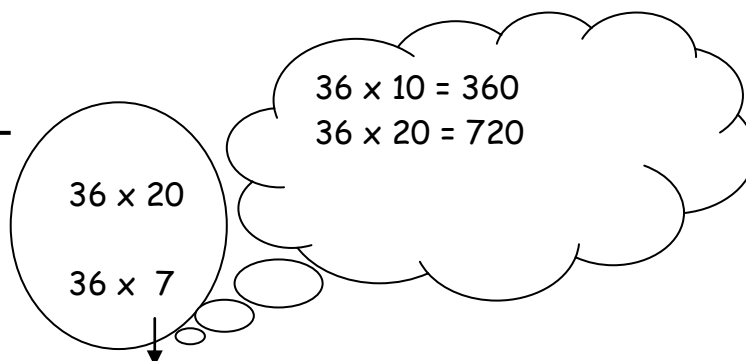
Children will continue to use written methods to solve short division $TU \div U$ and $HTU \div U$.

Long division $HTU \div TU$

$$972 \div 36$$

$$\begin{array}{r} 27 \\ 36 \overline{) 972} \\ \underline{- 720} \\ 252 \\ \underline{- 252} \\ 0 \end{array}$$

Answer : 27



Any remainders can be shown as fractions, i.e. if the children were dividing 32 by 10, the answer should be shown as $3 \frac{2}{10}$ which could then be written as $3 \frac{1}{5}$ in it's lowest terms.

If children are able to use the chunking method for long division, as set out above, by the end of Y6 this would be the ideal. The children will not be penalised for using this method during the KS2 SATs from 2016 onwards, providing they don't subtract the same chunk twice or more.

STAGE 12

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

$$87.5 \div 7$$

12.5		
7 $\overline{) 87.5}$	7 x 10	7 x 10 = 70 7 x 0.5 = 3.5
- 70.0	7 x 2	
17.5	7 x 0.5	
- 14.0		
3.5		
- 3.5		
0	↓	
Answer :	12.5	

Pupils may well struggle with the method set out above and the following formal compact method may well be more appropriate for them.

$$\begin{array}{r}
 12.5 \\
 7 \overline{) 87.5} \\
 \underline{70} \\
 17.5 \\
 \underline{14} \\
 3.5 \\
 \underline{3.5} \\
 0
 \end{array}$$

x ÷ x ÷ x ÷ x ÷ x ÷ x

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.