PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

Overview

This document is set out in stages rather than in year groups to take account for children's different rates of learning. The following extract comes from the Mathematics Programme of Study: Key Stages 1 and 2, September 2013.

The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study. In addition, schools can introduce key stage content during an earlier key stage, if appropriate.

Teachers need to be mindful of the need for each child to be taught the formal written methods for multiplication by the end of year 6.

For an overview of age related expectations for each year group, look at the attached progression map for x and ÷ produced by the NCETM.

Other key points

- New learning is likely to be taught to groups rather than the whole class to acknowledge the different learning stages of the children.
- Children need to understand that multiplication is commutative and use this information to rearrange calculations knowing that 4 x 6 = 24 gives the same answer as 6 x 4 = 24.
- Children need to understand that multiplication is repeated addition.
- Ensure that children understand the = sign means is the same as, not makes, and that children see calculations where the equals sign is in a different position, e.g. 3 x 5 = 15 and 15 = 3 x 5.
- Children should be encouraged to approximate before calculating and check whether their answer is reasonable.

USING AND APPLYING DIVISION FACTS

CHILDREN SHOULD BE ABLE TO UTILISE THEIR TABLES KNOWLEDGE TO DERIVE OTHER FACTS.

E.G. IF I KNOW 3 X 7 = 21, WHAT ELSE DO I KNOW?

30 X 7 = 210, 300 X 7 = 2100, 3000 X 7 = 21 000, 0.3 X 7 = 2.1 ETC

USE CLOSELY RELATED FACTS ALREADY KNOWN

13 X 11 = (13 X 10) + (13 X 1)

= 130 + 13 = 143

MULTIPLYING BY 10 OR 100

KNOWING THAT THE EFFECT OF MULTIPLYING BY 10 IS A SHIFT IN THE DIGITS ONE PLACE TO THE LEFT.

KNOWING THAT THE EFFECT OF MULTIPLYING BY 100 IS A SHIFT IN THE DIGITS TWO PLACES TO THE LEFT.

PARTITIONING

23 X 4 = (20 X 4) + (3 X 4)

= 80 + 12

= 102

USE OF FACTORS

8 X 12 = 8 X 4 X 3

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

MULTIPLICATION GLOSSARY

MULTIPLICAND (7): THE NUMBER BEING MULTIPLIED.

MULTIPLIER (8): THE NUMBER BY WHICH THE MULTIPLICAND IS MULTIPLIED.

PRODUCT (56): THE RESULT OF MULTIPLYING ONE NUMBER BY ANOTHER.

7 X 8 = 56

STAGE 1

It is expected that there will be lots of practical activities to support children's growing awareness and understanding of multiplication.

Children can complete practical activities involving grouping objects. Rhymes and stories can be used that involve counting in different intervals.

Use apparatus to sort objects into groups.

E.g. Sort six compare bears into 2 groups. How many in each group?

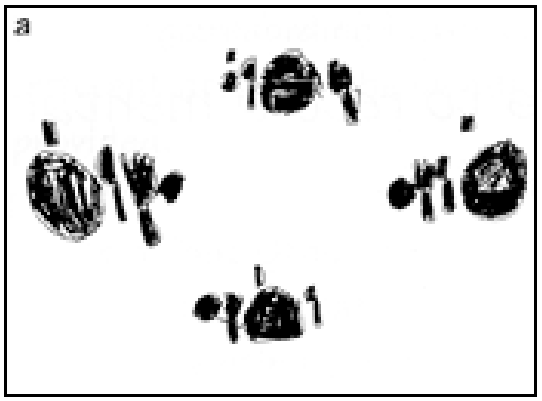
2 lots of 3
2 groups of 3

3 x 2

A mixture of pictures, words and symbols will be used by children in order to explain to someone else the methods that they have used.

STAGE 2

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities involving equal sets or groups using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.



STAGE 3

Children will use practical equipment to make groups of objects to represent multiplication. They should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

STAGE 4

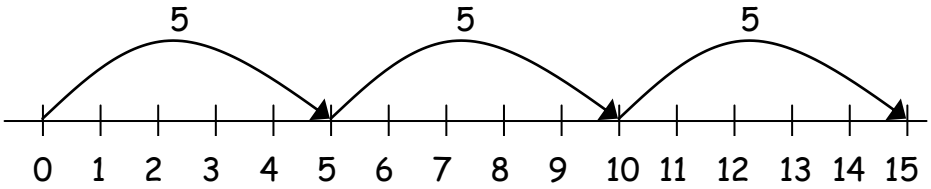
Children will develop their understanding of multiplication and use jottings to support calculation:

- ✓ **Repeated addition**

3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

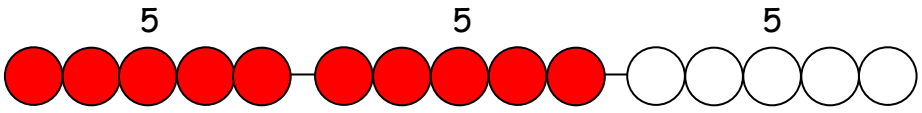
Repeated addition can be shown easily on a number line:

5 x 3 = 5 + 5 + 5



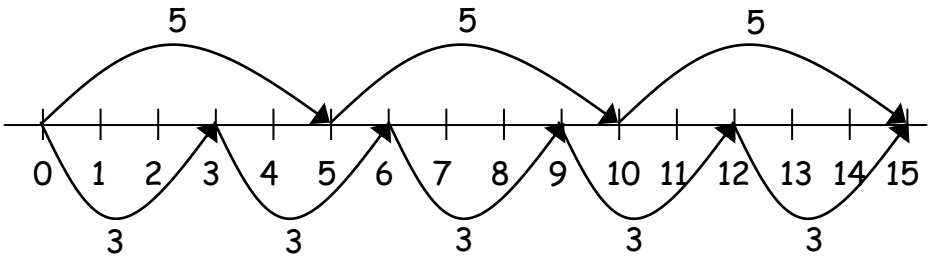
and on a bead bar:

5 x 3 = 5 + 5 + 5

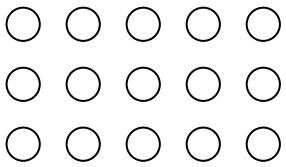


✓ **Commutativity**

Children should know that 5 x 3 has the same answer as 3 x 5. This can also be shown on the number line.



✓ **Arrays**



5 x 3 = 15

3 x 5 = 15

Children should utilise multiplication as repeated addition - linked to arrays **(as this knowledge will support with the development of the grid method)**. They should also use jottings to support their calculation. These should be supported by the use of counters/cubes.

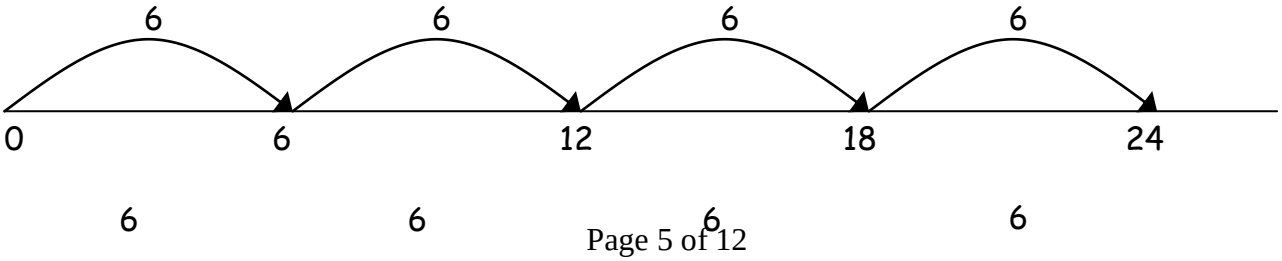
STAGE 5 Empty Numberline

Children will continue to use:

✓ **Repeated addition**

4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4

Children should use number lines or bead bars to support their understanding.



✓ **Scaling**

e.g. Find a ribbon that is 4 times as long as the blue ribbon

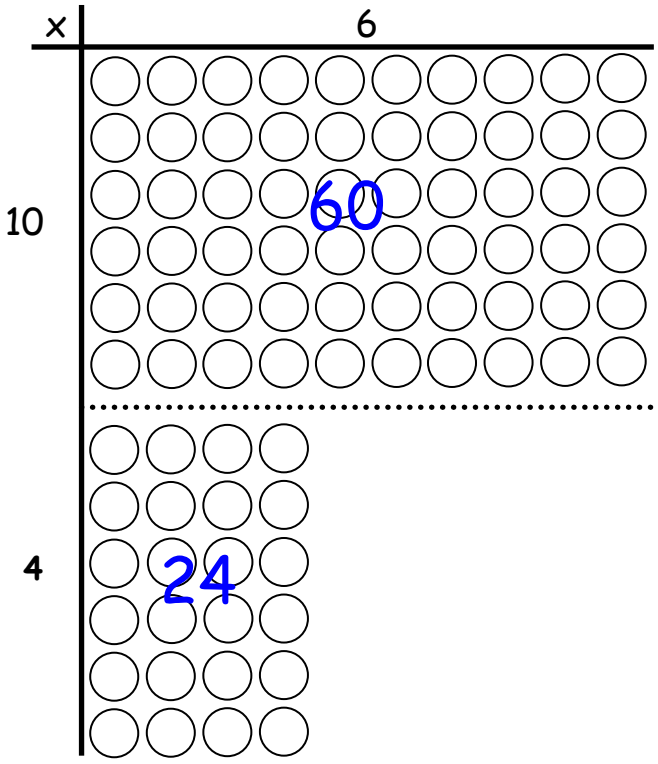
✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

□ x 5 = 20 3 x △ = 18 □ x O = 32

✓ **Partitioning**

38 x 5 = (30 x 5) + (8 x 5)
= 150 + 40
= 190

STAGE 6 (Grid method array form)



(10 x 6) + (4 x 6)

60 + 24

84

STAGE 7

TU x U (Short multiplication – multiplication by a single digit)

38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266

×	7
30	210
8	56
	266

$$\begin{array}{r} 210 \\ +\ 56 \\ \hline 266 \end{array}$$

STAGE 8 (Expanded Short Multiplication)

$$\begin{array}{r} 30+8 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$$

$$\begin{array}{r} 30\times 7=210 \\ 8\times 7=\ \ 56 \end{array}$$

$$\begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$$

STAGE 9 (Formal short Multiplication)

$$\begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 266 \\ {\scriptstyle 5} \end{array}$$

STAGE 10 (Extend to decimals)

7.8 x 8 =

x	8
7	56
0.8	6.4
	62.4

1

Leading to:

$$\begin{array}{r} 7.8 \\ \times\ 8 \\ \hline 62.4 \\ \hline \end{array}$$

6

STAGE 11

TU x TU
(Long multiplication – multiplication by more than a single digit)

Children will approximate first.

56 × 27 is approximately 60 × 30 = 1800.

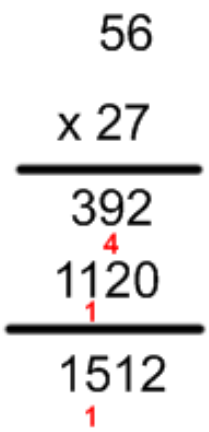
×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

	50	6	
×	20	7	
	1000	350	1350
	120	42	162
			1512
			1

Reduce the recording further

$$\begin{array}{r} 56 \\ \times\ 27 \\ \hline 1000 \\ 120 \\ 350 \\ 42 \\ \hline 1512 \\ 1 \end{array} \qquad \begin{array}{r} 50 \times 20 = 1000 \\ 6 \times 20 = 120 \\ 50 \times 7 = 350 \\ 6 \times 7 = 42 \end{array}$$

Leading to:

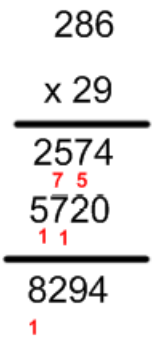


This method needs to be taught to all children by the end of KS2. Re. KS2 SATs from 2016 onwards, if this method was used by a child and he/she got the answer wrong but used this method they'd get a mark. This is providing they'd only made one mistake with their workings.

STAGE 12 (Extend to HTU bu TU)

×	20	9	
200	4000	1800	5800
80	1600	720	2320
6	120	54	174
			8294
			1

Leading to:



STAGE 13 (Extend to decimals).

	10	5	
20	200	100	300
4	40	20	60
0.5	5	2.5	7.5
0.06	0.6	0.3	0.9
			368.4
			1

Leading to:

$$\begin{array}{r} 24.56 \\ \times\ 15 \\ \hline 122.80 \\ {\scriptstyle 2\ 2\ 3}\ 245.60 \\ \hline 368.40 \\ {\scriptstyle 1} \end{array}$$

× ÷ × ÷ × ÷ × ÷ × ÷ × ÷ ×

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.