

PROGRESSION THROUGH CALCULATIONS FOR ADDITION

Overview

This document is set out in stages rather than in year groups to take account for children's different rates of learning. The following extract comes from the Mathematics Programme of Study: Key Stages 1 and 2, September 2013.

The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study. In addition, schools can introduce key stage content during an earlier key stage, if appropriate.

Teachers need to be mindful of the need for each child to be taught the formal column written method for addition by the end of year 6.

For an overview of age related expectations for each year group, look at the attached progression map for + and - produced by the NCETM.

Other key points

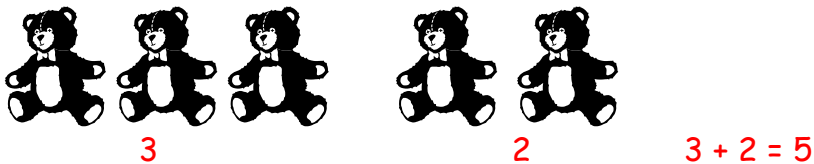
- New learning is likely to be taught to groups rather than the whole class to acknowledge the different learning stages of the children.
- Children should understand that addition is commutative and therefore calculations can be rearranged, e.g. $4 + 13 = 17$ is the same as $13 + 4 = 17$.
- Ensure that children understand the = sign means is the same as, not makes, and that children see calculations where the equals sign is in a different position, e.g. $3 + 2 = 5$ and $5 = 3 + 2$.
- Children should be encouraged to approximate before calculating and check whether their answer is reasonable.

STAGE 1

Counting, knowing the order of numbers, lots of practical activities with no written recording.

Combine two groups of objects and begin to record pictorially.

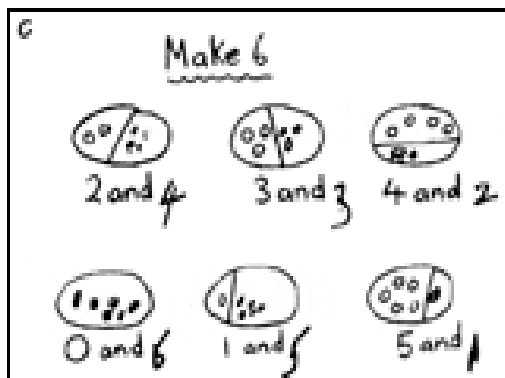
Eg, Jane has 3 bears. She was given 2 more. How many does she have now?



Children will use a mixture of words and symbols in order to explain to someone else the methods they have used.

STAGE 2

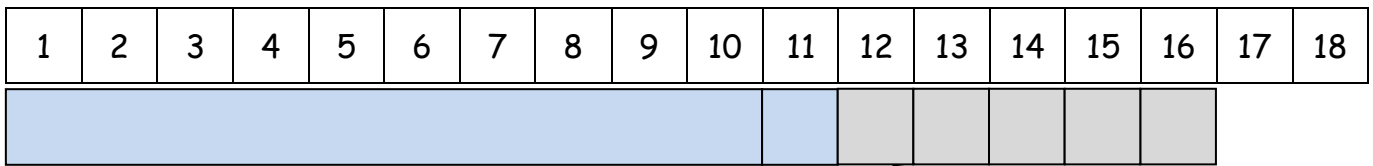
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



STAGE 3

Children will initially use practical equipment to combine groups of objects to find the total. They will move on to the use of number tracks and Base 10 equipment to support their developing understanding of addition. If possible, use two different colours of base 10 equipment so that the initial amounts can still be seen.

$11 + 5 =$



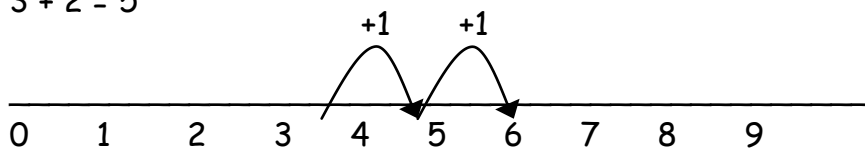
$11 + 5 = 16$

Model of Base 10 equipment

STAGE 4

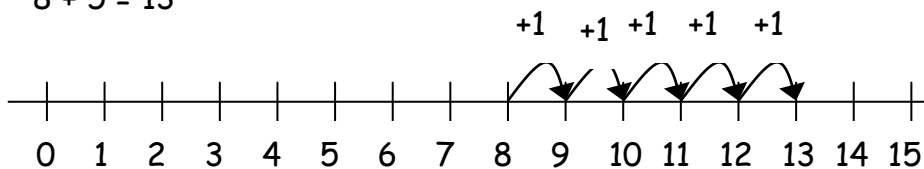
They use numbered number lines, and practical resources to support calculation and teachers *demonstrate* the use of the number line.

$3 + 2 = 5$

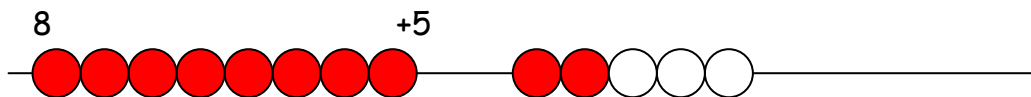


Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$8 + 5 = 13$



Bead strings or bead bars can be used to illustrate addition including bridging through ten, e.g. $8 + 5$ as $8 + 2 = 10$, then $10 + 3 = 13$. Cubes can also be used as an alternative.

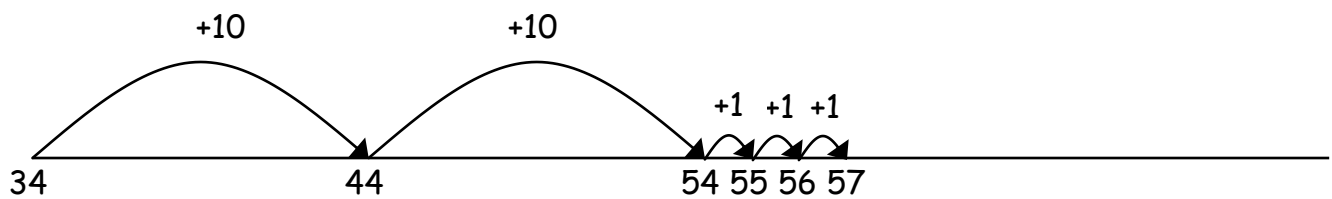


STAGE 5

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

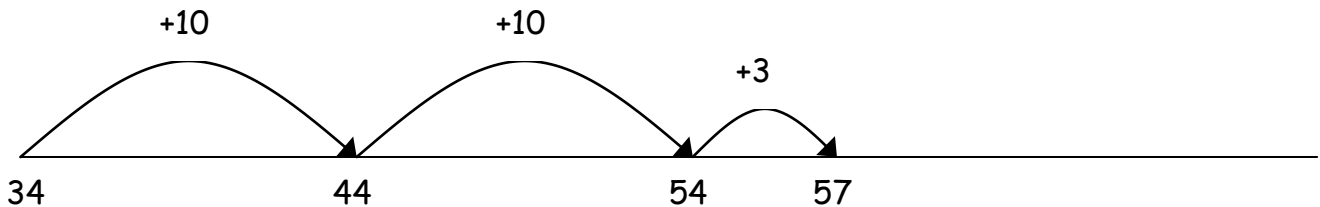
- ✓ First counting on in tens and ones.

$$34 + 23 = 57$$



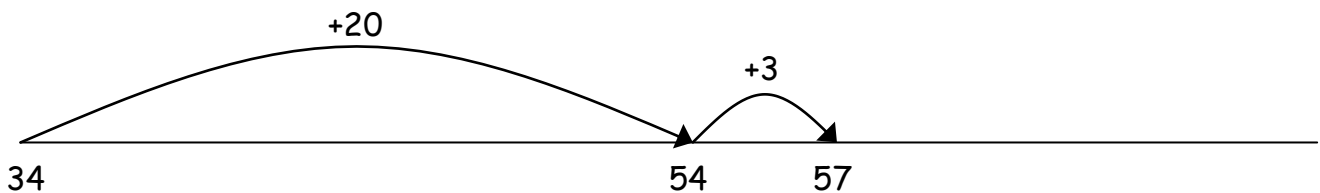
- ✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact $4 + 3 = 7$).

$$34 + 23 = 57$$



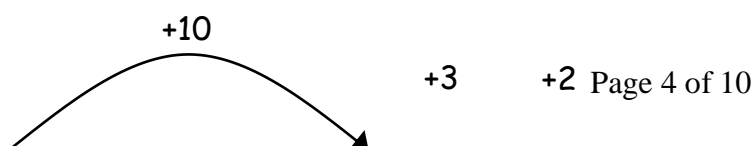
- ✓ Followed by adding the tens in one jump and the units in one jump.

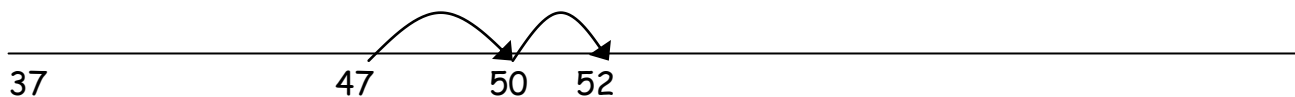
$$34 + 23 = 57$$



- ✓ Bridging through ten can help children become more efficient.

$$37 + 15 = 52$$



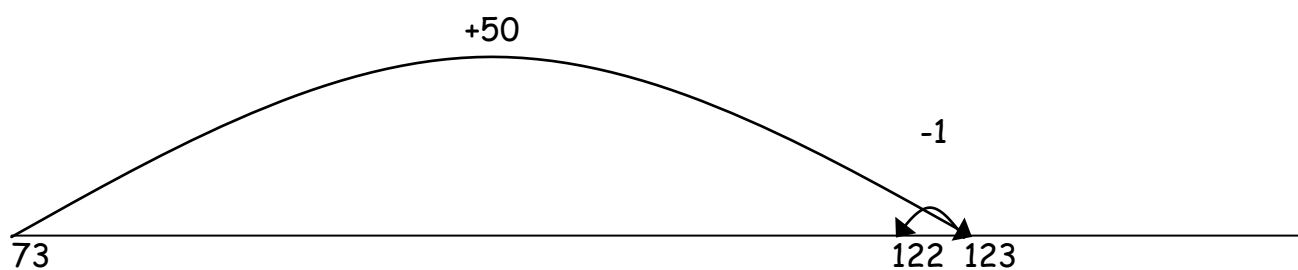


STAGE 6

Children will continue to use empty number lines with increasingly large numbers, including compensation (rounding up and then adjusting) where appropriate.

- ✓ Count on from the largest number irrespective of the order of the calculation.

$$49 + 73 = 122$$

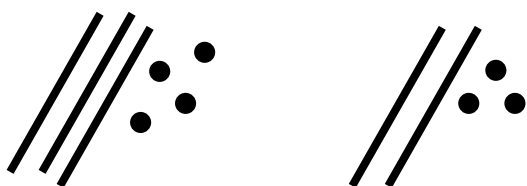


Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

STAGE 7

Children will pick up the use of Base 10 equipment again to support their calculations. They will record the calculations using their own drawings of the Base 10 equipment (as lines for the 10 rods and dots for the unit blocks)

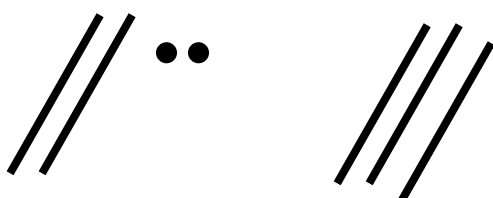
e.g. $34 + 23 =$

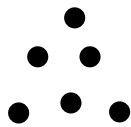
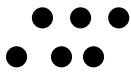


$$34 + 23 = 57$$

They would add the units first and then the tens before combining them.

e.g. $28 + 36 =$





$$28 + 36 = 64$$

When the units total more than 10, children should be encouraged to exchange 10 ones for 1 ten. This is the start of children understanding 'carrying' in vertical addition. PUPILS SHOULD PLAY SOME EXHANGING GAMES, PRIOR TO THIS STAGE (See attached games).

STAGE 8

The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums.

Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.

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Record steps in addition using partitioning:

$$47 + 76 = (47 + 70) + 6 = 117 + 6 = 123$$

Or by partitioning into specific tens and units

$$47 + 76 = (40 + 70) + (7 + 6) = 110 + 13 = 123$$

Partitioned numbers are then written under one another:

$$\begin{array}{r} 47 = 40 + 7 \\ + 76 = 70 + 6 \\ \hline 110 + 13 = 123 \end{array}$$

STAGE 9

During this stage, the pupils will experience carrying below the line.

It is more beneficial for children's understanding to go through the expanded methods of calculation (see below) as steps of development towards a formal written method. Children will build on their knowledge of using Base 10 equipment.

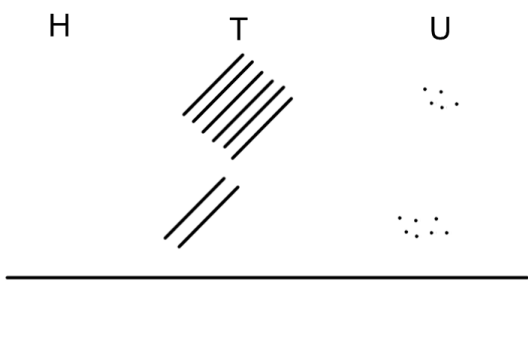
$$\begin{array}{r} 67 \\ + 24 \\ \hline 11 \text{ (7+4)} \\ \underline{80 \text{ (60 + 20)}} \\ 91 \end{array}$$

Children should add the least significant digits first (i.e. start with the units/ones),

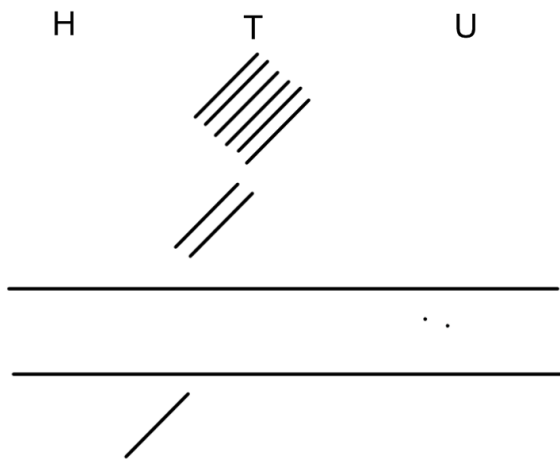
They can use a place value grid to begin to set the calculation out vertically, using Base 10,

and to support their knowledge of how units would transfer to tens. (as in Step 2 in the diagram below). e.g. $65 + 27$

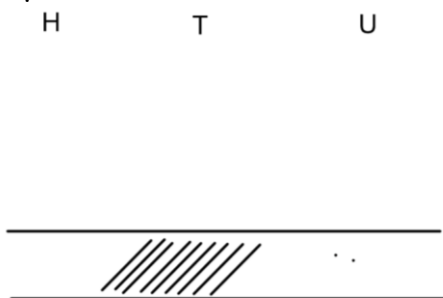
Step 1



Step 2



Step 3



STAGE 10

During this stage the pupils are taught the formal written method of column addition. Base 10 may still need to be used alongside this method to help children understand the underlying concepts.

$$\begin{array}{r} 783 \\ + 42 \\ \hline 825 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 367 \\ + 85 \\ \hline 452 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 321 \\ + 7 \\ \hline + 48 \\ \hline 376 \end{array}$$

$$\begin{array}{r} £3.48 \\ + £0.78 \\ \hline £4.26 \\ \hline 1 \quad 1 \end{array}$$

Using similar methods, children will:

- add several numbers with different numbers of digits;
- begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;
- know that the decimal points should line up under each other, particularly when adding mixed amounts, e.g. £3.59 + 78p.

STAGE 11

Children should extend the carrying method to numbers with at least four digits.

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \hline 111 \end{array}$$

$$\begin{array}{r} 3121 \\ + 37 \\ + 148 \\ \hline 3306 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 3.20 \\ + 2.88 \\ \hline 6.08 \\ \hline 1 \end{array}$$

Using similar methods, children will:

- add several numbers with different numbers of digits;
- begin to add two or more decimal fractions with up to three digits and the same number of decimal places;
- know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m + 280 cm.

STAGE 12

Children should extend the carrying method to number with any number of digits.

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ \hline 111 \end{array}$$

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ \hline 111 \end{array}$$

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ \hline 121 \end{array}$$

$$\begin{array}{r} 401.20 \\ + 26.85 \\ + 0.71 \\ \hline 428.76 \\ \hline 1 \end{array}$$

Using similar methods, children will

- add several numbers with different numbers of digits;
- begin to add two or more decimal fractions with up to four digits and either one or two decimal places;

- *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. $401.2 + 26.85 + 0.71$.*

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By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.