PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

Overview

This document is set out in stages rather than in year groups to take account of children's different rates of learning. The following extract comes from the Mathematics Programme of Study: Key Stages 1 and 2, September 2013.

The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study. In addition, schools can introduce key stage content during an earlier key stage, if appropriate.

Teachers need to be mindful of the need for each child to be taught the formal column written method for subtraction by the end of year 6.

For an overview of age related expectations for each year group, look at the attached progression map for + and – produced by the NCETM.

Other key points

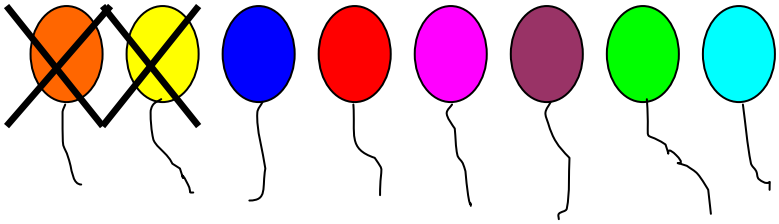
- New learning is likely to be taught to groups rather than the whole class to acknowledge the different learning stages of the children.
- Children should understand that subtraction is the removing or taking away one quantity from another (not necessarily the smaller number from the larger one) or finding the difference between two separate quantities.
- Children should understand that, unlike addition, subtraction is **not** commutative.
- Ensure that children understand the = sign means is the same as, not makes, and that children see calculations where the equals sign is in a different position, e.g. 9 – 5 = 4 and 4 = 9 – 5.
- Children should be encouraged to approximate before calculating and check whether their answer is reasonable.

STAGE 1

Counting backwards, knowing the order of numbers, lots of practical activities with no written recording.

Children need practical activities of taking away that is finding how many are left from a collection of objects when some are removed.

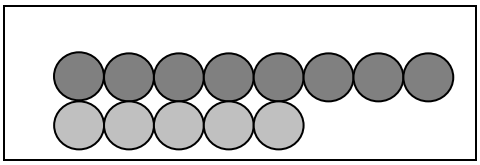
E.g. There were 8 balloons. Two popped. How many balloons are left?



8 – 2 = 6

Children also need practical activities around 'finding the difference', which involves making a comparison between the numbers in two groups of objects. They need to recognise that this is another example of subtraction.

E.g. How many more biscuits does Sally have than you? (The biscuits are represented by counters).

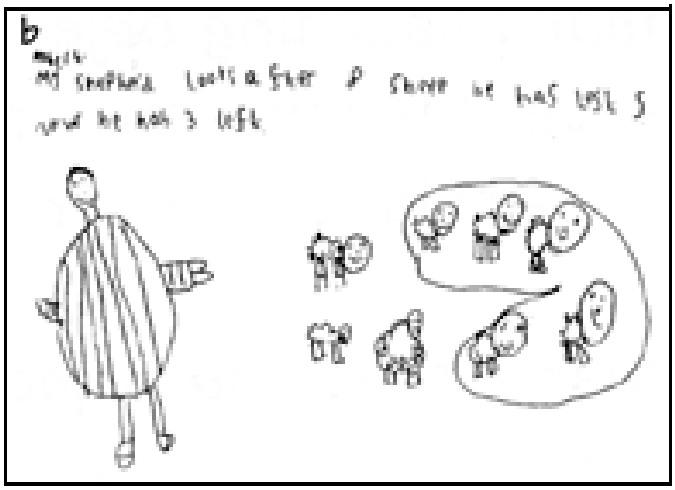


'Sally has 3 more than me'.

A mixture of words and symbols will be used by children in order to explain to someone else the methods that they have used. Children will use a variety of ways of recording subtraction, reflecting the mental methods used.

STAGE 2

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.



Children who are ready may record this as 8 – 5 = 3

STAGE 3

Children will use practical equipment for subtraction by taking away (counting back).

13 – 5 =

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

Count out 13 cubes along the number track followed by removal of 5 cubes to give answer:

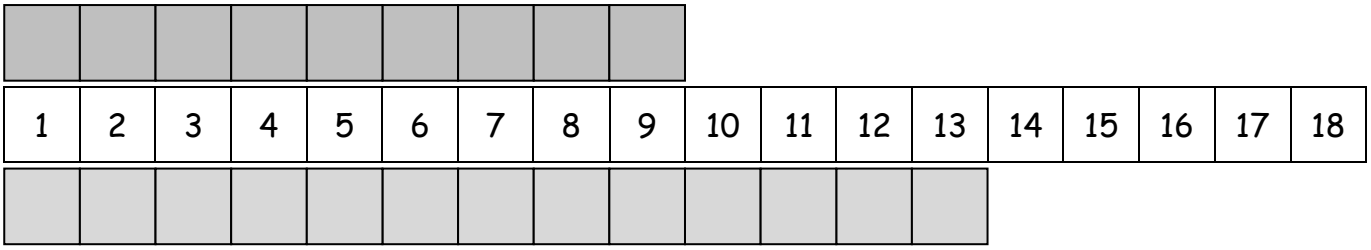
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

13 – 5 = 8 It is important that children keep track of how many have been removed.

Subtraction as finding the difference

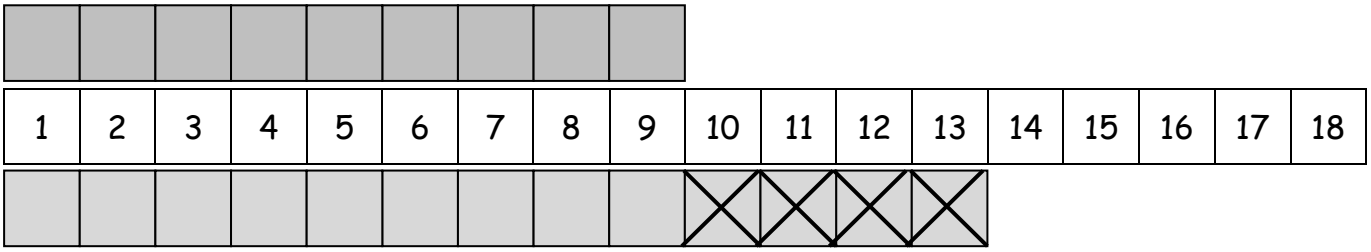
In preparation for understanding how to find the difference by counting up, children should be shown that finding the difference is linked to subtraction and the teacher should ensure the children know that it is an appropriate strategy to use when the numbers are close together.

e.g. 13 – 9



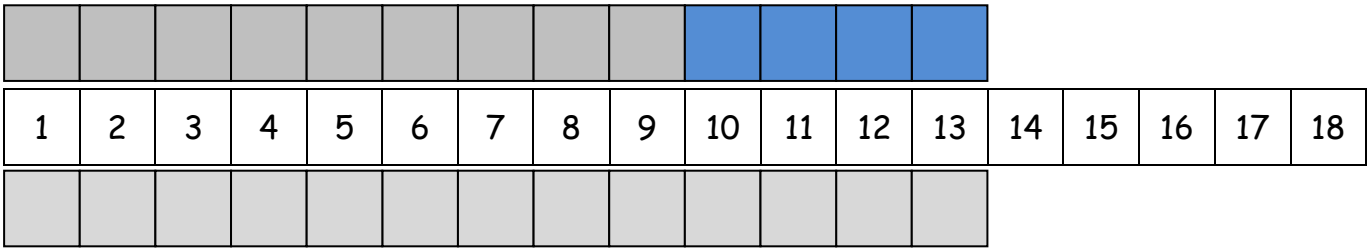
Children should use the Base 10 unit cubes and count out the correct amounts, placing one set above the number track and one below.

To find the difference, children need to identify how to make the two amounts the same. This should begin by removing cubes from the larger amount, one at a time, until it is the same size as the smaller amount. As each cube is removed the children count how many are being removed.



Children should understand that this calculation is 13 - ? = 9

The next stage of finding the difference regards making the smaller amount the same size as the larger amount by counting on.



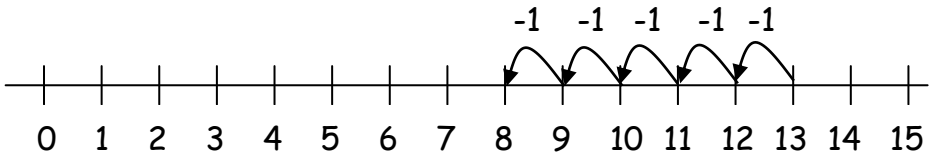
Children should understand that this calculation is 9 + ? = 13

NB – It is useful to present the context of difference in real life contexts such as comparing two measurements or when interpreting block graphs.

STAGE 4

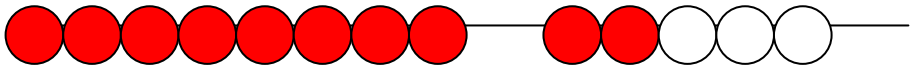
Children then begin to use numbered number lines to support their own calculations - using a numbered number line to count back in ones.

13 – 5 = 8



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

13 – 5 = 8



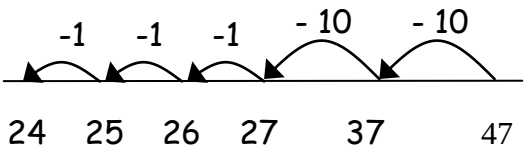
STAGE 5

Children will begin to use empty number lines to support calculations.

Counting back

✓ First counting back in tens and ones.

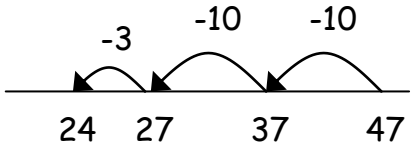
47 – 23 = 24



Then helping children to become more efficient by subtracting the units in one jump (by using the known fact 7 – 3 = 4).

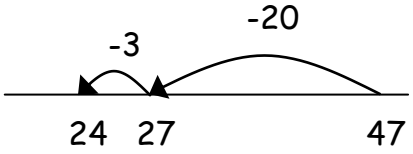
✓

47 – 23 = 24



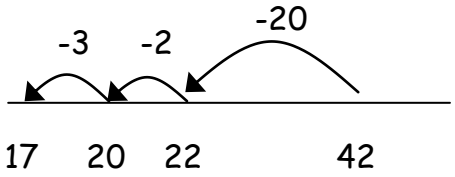
✓ Subtracting the tens in one jump and the units in one jump.

47 – 23 = 24



✓ Bridging through ten can help children become more efficient.

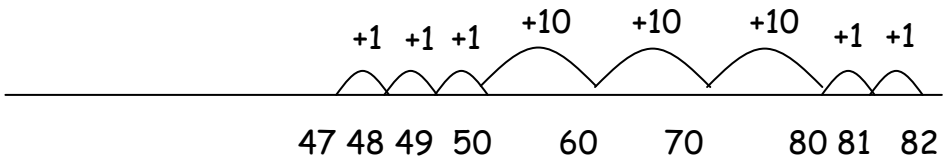
42 – 25 = 17



Counting on

Counting on will build towards an efficient and reliable subtraction method that allows children to keep track of their recorded steps.

82 – 47 counting up to the next ten from 47, then counting in 10's and then in jumps of 1.



Help children to become more efficient with counting on by:

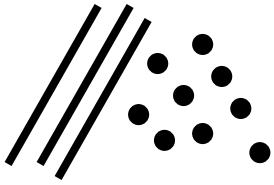
✓ Counting to next 10 in one jump;
✓ Counting on the tens in one jump and then the units in one jump;

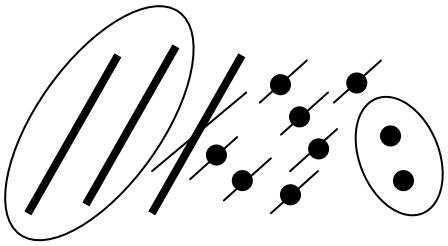
Children will continue to use empty number lines with increasingly large numbers.

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

STAGE 6

Children will move on to using the Base 10 equipment to support their calculations. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount.
e.g. 39 - 17 =





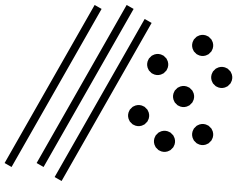
Children would count out 39 using the Base 10 equipment (3 tens and 9 units) and would remove 7 units and then one ten, counting up the answer of 2 tens and 2 units to give 22.

Circling the tens and units that remain will be modelled by the teacher but does not have to be written by pupil in their answer.

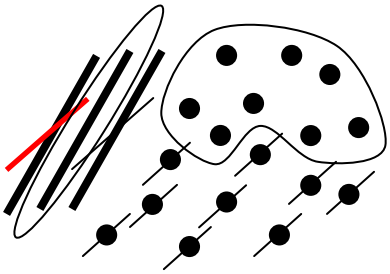
PUPILS SHOULD PLAY SOME EXHANGING GAMES, PRIOR TO THIS STAGE (See attached games).

When exchange is required:

37 - 19 =



Children can see that there are not enough units available to subtract 9 units so they need to exchange a ten for 10 units. This will become:



Children would count out how many tens and units are left to give the answer (18).

Children will be encouraged to record this by drawing representations of the Base 10 material and crossing out those pieces that they are subtracting. If children are representing exchange, they should be encouraged to cross out a 10 rod line in a different

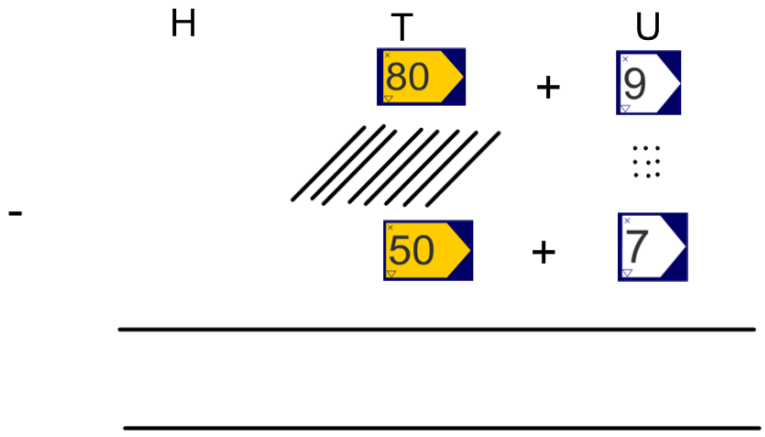
colour (red in the example above, to avoid confusion between the exchange and the subtraction) and replace with 10 unit dots.

STAGE 7

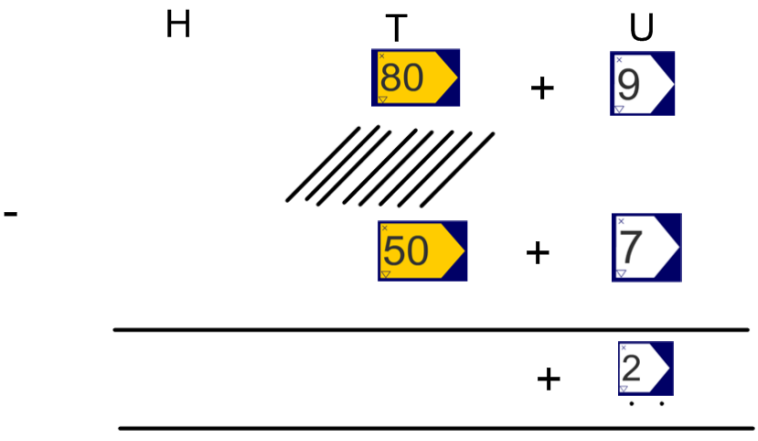
Children should begin the method of expanded layout subtraction column method with, initially, TU – TU calculations not involving any decomposition. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to show the decomposition of the number. This could be undertaken on a place value mat.

When solving the calculation 89 – 57, children need to understand that the number being subtracted (57) does not appear as an amount on its own, but rather as part of the larger amount. Therefore, when using Base 10 materials, children would need to count out only the 89.

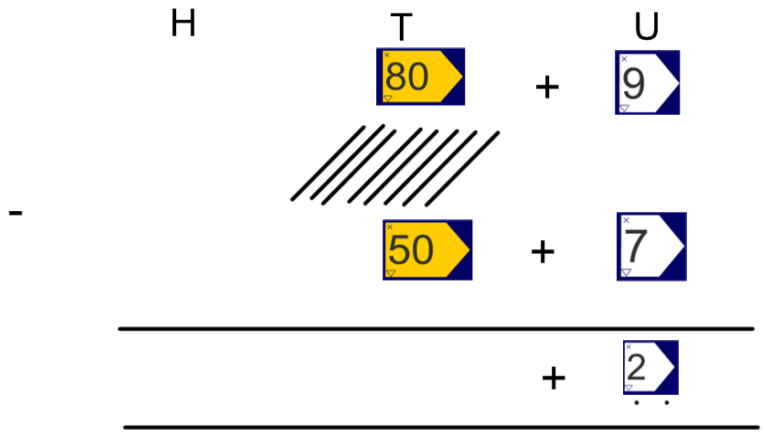
Step 1



Step 2

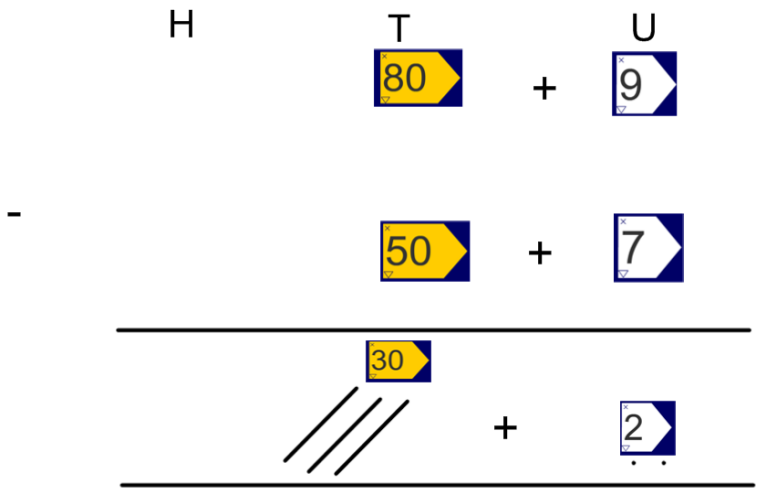


Step 3

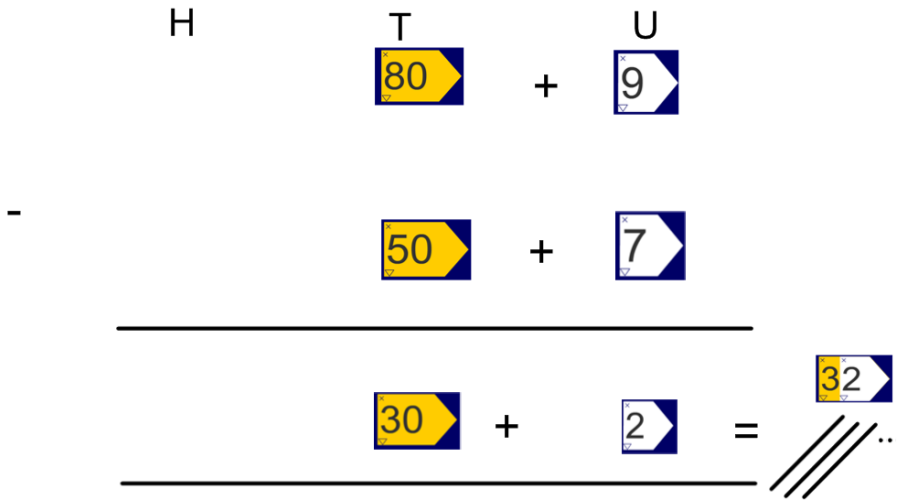


80 subtract 50

Step 4



Step 5



Leading to:

80 + 9

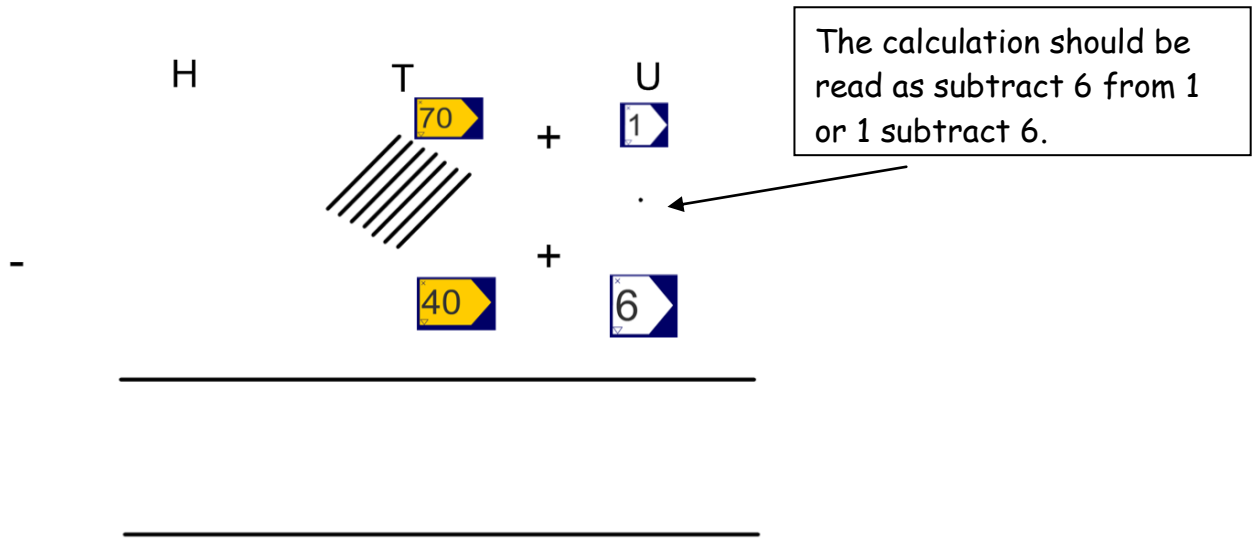
\- 50 + 7

30 + 2 = 32

From this the children will begin to solve problems which involve exchange (Decomposition):

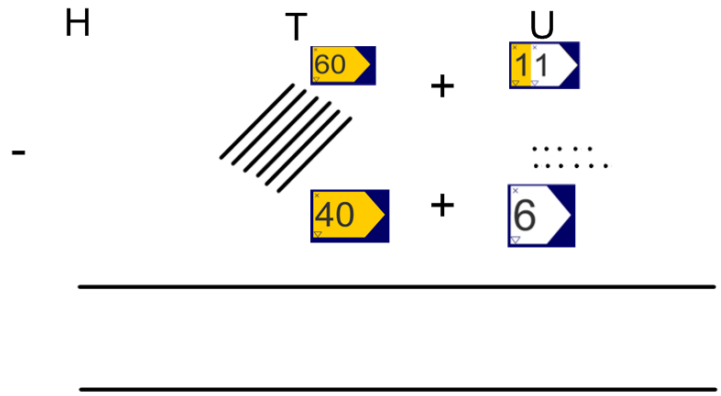
Example 71 – 46 =

Step 1



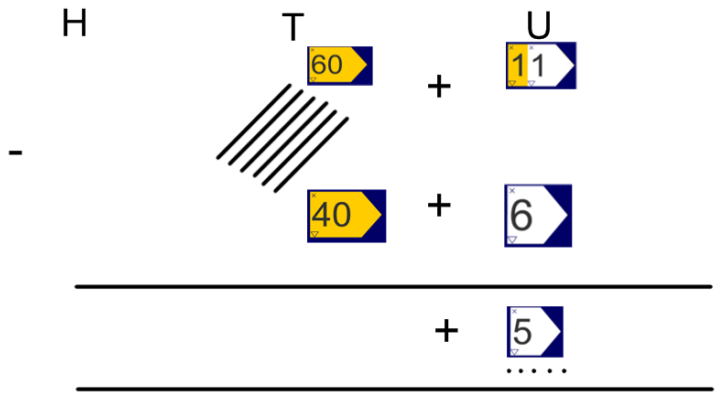
Children can see that there are not enough units to subtract 6 units so they need to exchange a ten for ten units. This will become:

Step 2

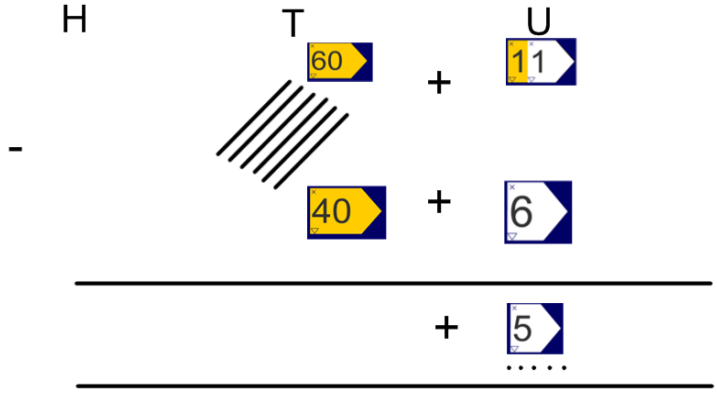


11 subtract 6

Step 3

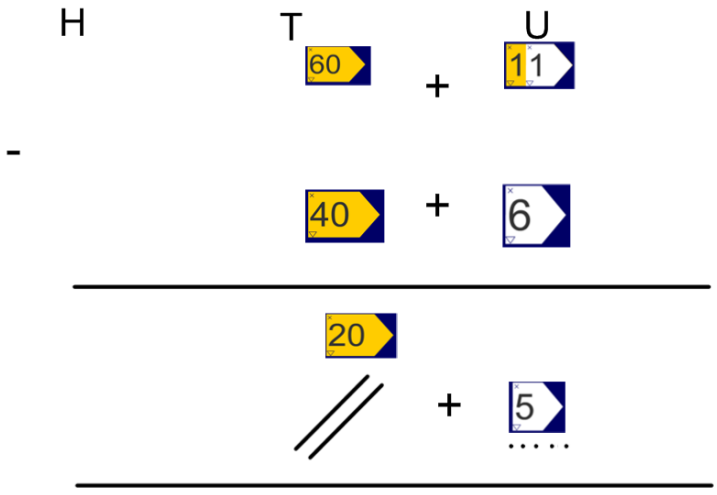


Step 4

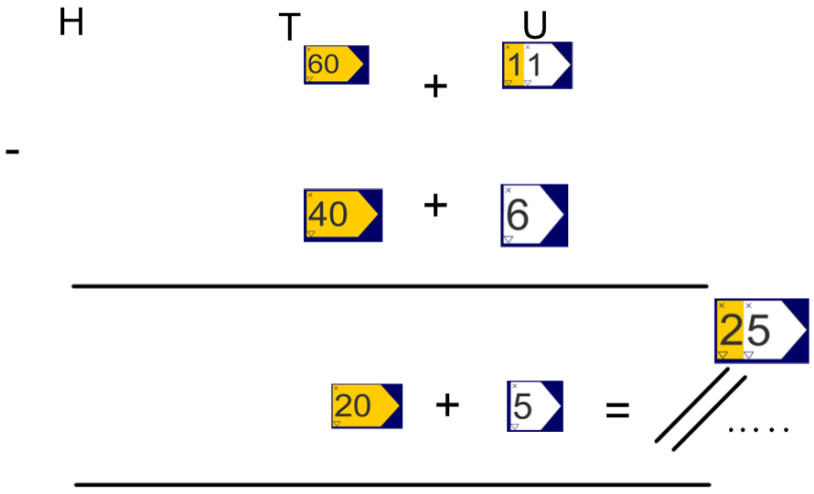


60 subtract 40

Step 5



Step 6



Leading to:

This would be recorded by the children as

$$\begin{array}{rrrrl} & \overset{60}{\cancel{7}}0 & + & {}^{1}1 & \\ - & 40 & + & 6 & \\ \hline & 20 & + & 5 & = 25 \end{array}$$

Children should know that units line up under units, tens under tens, and so on.

STAGE 8 (Expanded HTU - TU)

Children should know that units line up under units, tens under tens, and so on.

754 =
- 86

Step 1 700 + 50 + 4
- 80 + 6

Step 2 700 + 40 + 14 *(adjust from T to U)*
- 80 + 6

Step 3 600 + 140 + 14 *(adjust from H to T)*
- 80 + 6
600 + 60 + 8 = 668

This would be recorded by the children as

600 140
~~7~~~~0~~~~0~~ + ~~5~~~~0~~ + ^{1}4
- 80 + 6
600 + 60 + 8 = 668

STAGE 8 cont. (COMPACT METHOD)

Leading to: -

614 1
~~7~~~~5~~4
- 86
668

This can then be extended to numbers with any number of digits subtract numbers with any number of digits.

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;*
- ✓ *know that decimal points should line up under each other.*

For example:

£8.95 = 8 + 0.9 + 0.05

Stage 8

Leading to: -

-£4.38 - 4 + 0.3 + 0.08

= 8 + 0.8 + 0.15 *(adjust from t to h)*

- 4 + 0.3 + 0.08

4 + 0.5 + 0.07

= £4.57

8 1

£8.95

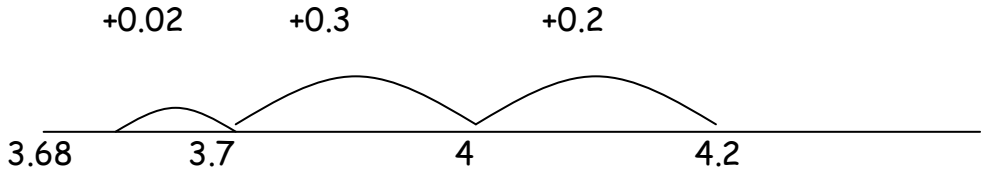
- £4.38

£4.57

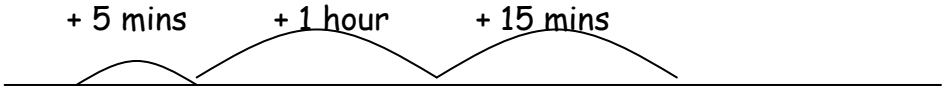
The Use of The Empty Number line at Any Stage of Development

The empty number line is useful to use at any stage of development for the following areas: -

Decimals e.g. 4.2 – 3.68 = 0.52

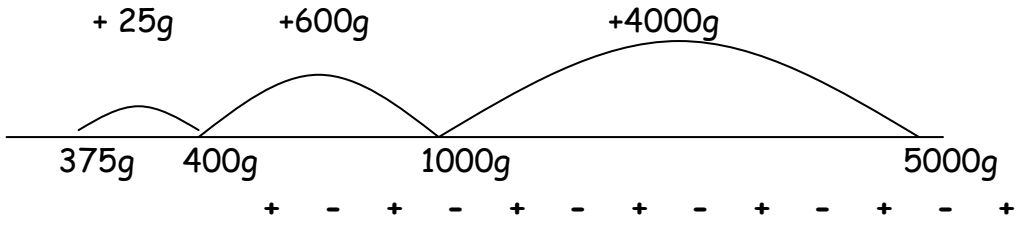


Time e.g. find the difference between 04.55am and 06.15am



04:55am 05:00am 06:00am 06:15am

Conversion in measure – capacity, length etc. e.g. 5000g – 375 g = 4625 g



By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods